

## PART 3: INTRODUCTION TO EACH PUZZLE TYPE

One of the challenges that puzzles present to elementary students is their messiness. They often resist the neat-niks' attempts to proceed from left to right and top to bottom, filling in each answer as they go along. Some gifted children are especially distressed by the need to move on before getting the complete answer, while others delight in the freedom of skipping around. You might need an extra dose of patience with the first group, but once they experience the pleasure of completing a puzzle, they will be more willing to work outside their box.

### BY THE NUMBERS

I adapted these puzzles from others that I'd seen in teaching materials. In those sources, they were primarily missing addend exercises that did not require any higher level thinking skills. I've intentionally made them more challenging. I also added the rule that if a number appears in the puzzle, none of the letters in the puzzle will represent that number. These were one of the first types of puzzles that I made for Dell Puzzle Publications' magazines.

These puzzles are especially appropriate for developing flexibility in using the inverse operations of addition and subtraction and for discussing fact families. Students should be encouraged to think of the top number in the subtraction puzzles as the sum of the other two numbers. Those two numbers would then be thought of as addends.

### LOST SUMS

As far as I know, Dell is the only publisher of these puzzles. I have been constructing them for Dell for about 20 years. The puzzles here follow the same format as those that I have sold to Dell, but include additional clues to make them appropriate for elementary solvers.

These puzzles can be used to extend students' understanding of place value. In most of the puzzles, it will be necessary to analyze the need for regrouping in each place. For example, if the ones digit in the sum is a 4, will it represent a total of 4, 14, or 24? The lowest possible sum of three different counting numbers is 6, so it can't be 4. The only way it can be 24 is if the three addends are 7, 8, and 9. If any of these three digits appears elsewhere in the puzzle, or if one of the digits is given and it is lower than 7, then the only possible sum will be 14. This regrouping will need to be taken into account when determining the missing addends in the tens column.

As an extension activity, ask the children to find the sum of the three numbers given on the right-hand side of the puzzle. It will always be 45 because it is the sum of the digits 1 through 9. Then have them find the sum of the digits in the puzzle's sum. It will always be a multiple of 9 because it is the same as adding the digits 1 through 9 with some regrouping. Any series of nine consecutive numbers will add up to a multiple of 9. Will this be true of any arithmetical series of nine numbers, like nine consecutive odd numbers? What about a series of five numbers, or three, or six?

### PICTURE PUZZLES

These popular puzzles can be found in GAMES magazine and on the internet under various names, including "Paint by Number" and "Nonogrids." While I have constructed them and used them in gifted summer classes, I have not attempted to sell mine to either of those sources. The online puzzles are often computer generated and therefore do not produce a picture when completed.

Some of the gifted children that I've worked with have been resistant to spoiling the appearance of the final picture by inserting dots or circles in the spaces that they have determined must be empty. But those dots or circles are essential to solving the puzzles. For students whose perfectionistic tendencies may pre-

vent them from finding the solution, I would recommend giving them two copies of each puzzle. They can mark up one copy as needed and enter only the filled spaces in the second copy, thus preserving a more pristine appearance.

While the basic math required may seem too simple for gifted children, the reasoning can be rather difficult. Counting the squares individually can be done to determine which ones to fill in, but the students should be encouraged to look for patterns and to use addition and subtraction instead.

As an extension activity, ask the children to construct ratios, proportions, or fractions to compare the number of spaces to be filled in to the total number of spaces remaining in a given row or column. How large does that fraction need to be in order to determine which space or spaces must be filled in? Can they formulate an appropriate rule?

## CROSS NUMBERS

Cross Numbers appear in a variety of puzzle magazines and math puzzle websites. In some sources, one or more digits can be used more than once in a puzzle, while others do not appear at all. The list of numbers is then given below each puzzle. I've chosen to use each of the digits from 1 to 9 exactly once in each puzzle. This makes the puzzles a little easier for elementary solvers.

Cross Numbers do not follow the "order of operations" rules, but they could be used to begin a discussion of those rules. After a puzzle has been solved, ask the children how the answers would vary if they used those rules.

## KENKEN

These puzzles can be found online and in puzzle books and magazines. They are a variation on the popular Sudoku, in that every row and every column contains each digit exactly once. But Sudoku is a logic puzzle, with no math required, while KenKen includes the use of mathematical reasoning. Most KenKen puzzles are 6 x 6, especially those intended for children, but they can be found in other sizes as well.

As an extension activity, students can explore the relationship between odd and even addends and factors. If the sum of two numbers is odd, what does that tell you about the addends? What about the sum of three numbers? What do you know about the product if the quotient is even?

## ADDITION LOGIC

This is one of my favorite puzzles in the puzzle magazines that I buy. Those resources provide only five words in the list. I've increased the number of words to nine to adapt them to an elementary level. I've only seen them in Dell puzzle magazines, but they could be available from other sources as well.

Students might enjoy creating their own Addition Logic puzzles. It's not as easy as making a list of words that share the same nine letters. They will have to consider factors such as which letters should have the highest or lowest values in order to make the puzzles solvable. They can then try solving each other's puzzles, or make their own puzzle magazine to share with friends and family members.

## KAKURO

Contrary to popular belief, this puzzle was not invented in Japan. It was invented by an American and first published in Dell puzzle magazines in the 1950's under the name "Cross Sums." For many years, Dell's only supplier was a woman. But Japan is known for its love of math and logic puzzles, such as Sudoku (which was also invented in the United States and called "Number Place"), while Americans have always preferred crosswords and other word puzzles. Why is that?

It's impossible to make a crossword puzzle using Japanese characters. In a crossword, the words must be broken down into small units such as individual letters in order to form a grid of intersecting words. Japanese characters each contain a great deal more information than a single letter of the alphabet. Their words cannot be broken down into enough small units to make a crossword puzzle. Other types of word puzzles

such as Anacrostics present similar problems. So the Japanese have developed an interest in math and logic puzzles instead.

Kakuro is probably the most popular math puzzle in the world. They can be found in many books, magazines, and websites. Several variations are also available. Some publishers of Kakuro puzzles include a list of the combinations of addends for each possible sum. But that turns it into a logic puzzle. The mathematical reasoning comes in when the children have to figure out for themselves, for example, how many combinations there are of three different one-digit numbers that add up to 21. As your students successfully solve more and more Kakuros, they will remember more and more of the combinations and it will become more of a logic puzzle. But in the beginning, they should be required to find those combinations on their own.

As an extension activity, start a discussion on variations on the Kakuro rules. What if 0's were allowed? Would that make the puzzles easier or harder? What if you could repeat a digit in an answer? What if the puzzles used multiplication instead of addition? In that case, would you want to allow repeated digits?

## BUZZIPPERS

I invented these puzzles based on the number game “Buzz Zip,” which we used to play in elementary school math classes when I was a child. I don't know if teachers use the game anymore, but you can teach it to your class when you introduce the puzzles.

In the game, the students count one by one, beginning with “1,” and continuing around the classroom repeatedly until all the children except one have been eliminated. A target number is chosen before the game begins. For each player's turn, if the number to be named is a multiple of the target number, she says, “buzz” instead of the number. If the number contains the target number as a digit, she says, “zip.” If both of these conditions are true, she says, “buzz zip.” If she makes a mistake, she is eliminated. For example, if the target number is 3, the counting would go: “1, 2, buzz zip, 4, 5, buzz, 7, 8, buzz, 10, 11, buzz, zip, 14. . .” (In this case, navigating the 30's will be interesting!)

The counting continues as high as the class can go until all but one student have given the wrong response and thus have been eliminated. In recent years, when I have played this game with gifted children, those who are “out” have enjoyed following the play to try to catch the next mistake, thus keeping the entire group actively involved in the game.

The most obvious extension activities for these puzzles would be related to divisibility rules. In my first years of teaching elementary gifted math, I created the following poem to help my students remember those rules:

*2: I may be small, but I'm powerful, you'll see.  
Every even number is divisible by me.*

*3: My rule is simple and simply divine.  
Sum the digits of the number to get 3, 6, or 9.*

*4: Millions, billions, zillions or more – most of the numbers can be ignored.  
Just look at the last two digits – are they divisible by 4?*

*5: If you're looking for something easy, I'm your hero.  
When it's divisible by me, a number ends in 5 or 0.*

*6: There's really nothing unique about me.  
A number divisible by 6 must also be divisible by 2 and by 3.*

8: *Throw away the thousands and above, don't hesitate.  
The last three digits must be divisible by 8.*

9: *I'm similar to 3, but a little more refined.  
The sum of the digits must add up to 9.*

After reading the poem, every gifted child wants to know if there's a rule for the 7's. There is, but it's almost easier to do the division. For those who are overcome by curiosity, here's the rule: Multiply the ones digit by 2, then subtract it from the remaining digits. Continue doing this until the number is small enough to recognize whether it's a multiple of 7. Try, for example, 5492.  $549 - 4$  (ones digit  $\times 2$ ) = 545.  $54 - 10$  (new ones digit  $\times 2$ ) = 44. Since 44 isn't divisible by 7, 5492 isn't either. The reason this works is because it results in subtracting multiples of 21 (a multiple of 7) from the initial number. Somehow I could not compose a brief rhyme for this one!

### TRANSFORMERS

These puzzles are also known as "Word Arithmetic." I haven't seen any online, but they are popular in puzzle magazines. As the name implies, most sources produce puzzles in which the quotient, divisor, dividend, and 10-letter solution are all words or phrases. I've chosen not to use words since these puzzles are for the gifted population. Having worked with gifted children, I'm certain that some of them would spend hours trying to find the solution by rearranging the letters instead of doing the math.

To explore the inverse nature of multiplication and division, ask the students if they can transform the puzzle into a multiplication problem. Where would the divisor and quotient go? What about the remainder?

### TRISQUARES

I literally dreamed up these puzzles. I'd been spending a lot of time making puzzles, to the point that I was even dreaming about them. One night, I dreamed I was solving a puzzle. As I was waking up, I realized that I might be able to make a new type of puzzle like the one in my dream, so I made sure that I remembered how it worked. I had to do a little tweaking when I tested it out during the day, but TriSquares were the end result.

As an extension activity, have the students calculate the Least Common Multiple of the digits 1 through 9. Would they have to multiply all nine digits? What shortcuts could they use? How will they know when they've found the very lowest number that's divisible by all nine digits? Will this Least Common Multiple be evenly divisible by each of the products given in each puzzle? Why or why not?