INTRODUCTION

This book was written for several reasons. Primarily, it was written to answer questions related to how best to serve intellectually advanced mathematicians. The book is based on a series of questions which are conveniently posed as headings. The hope is that this book will help in-service teachers, parents, graduate students, and aspiring gifted educators such as pre-service teachers, adequately meet the needs of intellectually advanced mathematicians.

➢ Why another book on mathematical abilities?

The focus is on intellectually advanced mathematicians in grades K-6.

Frankly, there have not been many books related to mathematically gifted students. Further, mathematical gifted students are not the sole focus of this book as will be detailed shortly. The focus is on intellectually advanced mathematicians in grades K-6. Fortunately, of the books published, most have been published recently, and they have been done by prominent experts in the field of gifted education (e.g. Susan Assouline, Linda Sheffield, et cetera). Certainly, there have been some studies conducted on mathematical giftedness; the most salient of which are certainly Julian Stanley's (1975) innovative work called the Study of Mathematically Precocious Youth (SMPY) and V. A. Krutetskii's (1976) work on mathematical abilities in schoolchildren. Stanley's work focused on identifying gifted students with standardized assessments and Krutetskii's work focused on cognitive characteristics of advanced students. Nevertheless, it would be incumbent of anyone undertaking such a venture to mention the significant contributions of previous scholars. In recent years, several books have been written related to mathematics education of the gifted. These books are:

- *Developing mathematical talent: A guide for challenging and educating gifted students* (Assouline & Lupkowski-Shoplik, 2003).
- *Developing mathematically promising students* (Sheffield, 1999).
- *Encouraging your child's mathematical talent: The parent's guide* (Bossé, M., & Rotigel, 2006).
- *Extending the challenge in mathematics. Developing mathematical promise in K-8 students* (Sheffield, 2002).
- *Maths challenge* (Gardiner, 2000).
- *Math education for gifted students* (Johnsen & Kendrick, 2005).
- *Teaching mathematics to able children* (Koshy, 2001).
- *The peak in the middle* (Saul, Sheffield, & Assouline, 2010)

➢ What is the purpose of this book, and why is it different from others?

The primary purpose of this book is to maximally prepare and educate those involved in serving or raising intellectually advanced mathematicians. The operative term in the past sentence is 'maximally.' Currently, intellectually advanced mathematicians are served with all sorts of approaches, though few approaches may have the best interests of intellectually advanced students in mind. Many approaches are used in an attempt to save money and/or as a result of a lack of understanding of how to best serve intellectually advanced mathematicians. To best enable individuals concerned with intellectually advanced mathematicians the opportunity to serve them, this book has been written. Specifically, this book has been created to provide a mix of theoretical and pragmatic suggestions regarding how learning, not just teaching, can be maximized.

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A secondary purpose of this book is to provide a resource for those who feel they cannot afford to serve mathematicians of advanced intellect. To those who suggest that they cannot afford to serve such students, one should question whether they can afford NOT to serve such students. Intellectually advanced mathematicians may be the most precious commodity currently available to society. Neglecting them does no favors to society and, in fact, not educating these students will now cost more in the long run. This is the case because all societies need advanced mathematicians. Those who choose not to serve them currently will force others, i.e. in industry and government, to create, identify, or recruit them from another society. Hence, investing a few scant dollars in a book that will provide resources regarding how to challenge and adequately educate such students is a paltry, yet invaluable, investment. It is no secret that funding for gifted education will continue at its current abysmal rate or be reduced altogether.* (See page 10) To that end, interested parties may be asking, what can be done to adequately harness the potential of truly intellectually advanced mathematicians to help them develop into adults? Intellectually challenging oneself with this book is certainly a start, though exemplary teachers realize that professional development never stops when the end result is increased learning opportunities for students.

➢ Why the term intellectually advanced mathematicians rather than another term?

This is yet another great question and understanding the perspective of the author is paramount to understanding the direction of the book. Perhaps a more appropriate question might be, "Is there a perfectly appropriate term for those advanced in mathematics?" The answer is no; however, some terms are more appropriate than others and

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appropriate use of terms is contingent upon the needs of interested parties. The caveat issued is that this book will be helpful for individuals who serve any one of the forthcoming types of students. However, at the heart of this book is the focus on intellectually advanced students. To understand what the author means by the term intellectually advanced mathematicians, a look at past terms may be helpful to readers. Subsequent to the discussion of terminology, a rationale is provided for why the term intellectually advanced students has been adopted for this book.

Historically, many terms have been used to describe those with advanced capabilities. Terman, for instance, preferred the terms *gifted* (1925) or *genius* (1926) to describe folks he considered advanced (Jolly, 2008). He was also known to use the term talented. These terms started a very important discussion about individuals with advanced capabilities, but the terms subsequently became too generic, and they ultimately came to have a negative stigma as elitist. Not surprisingly, use of the terms suggested advancement in capabilities to fare well on the very test that Terman was instrumental in creating (i.e. the Stanford-Binet intelligence test). Later, the term *precocity* was used, but this term may have implied the simple notion of advanced development. Advanced development does not necessarily allude to sustained advanced capabilities or performance. It is important to note that these terms were used in reference to capabilities in general and not to mathematics in specific.

By the late 1950s and early 1960s, Krutetskii (1976) had used the term *mathematically capable* to define those who had exhibited advanced abilities in mathematics. This term alluded to the notion that the abilities existed, but it did not truly capture the essence of those who had harnessed such abilities. It may have been too narrow of a term and potentially implied that many students had high abilities but had not necessarily refined abilities in performance. Some 40 years later, Sheffield (1999) popularized the term *mathematically promising* in her book, and it is a comprehensive term though it has some similarities to Krutetskii's term mathematically capable. Though the term *promising* had been used previously, with respect to other abilities, in this case it was used in reference to students of promise in mathematics.

A final term worthy of mention is *academically advanced* (mathematicians). The term academically advanced mathematicians would, on the surface, seem akin to intellectually advanced mathematicians. This term is helpful, but it speaks of students who have effectively learned to play the school game. More specifically, academic advancement may allude to *students who have simply recorded high marks in mathematics or to students who are capable of regurgitating mathematical procedures on assessments.* It does not therefore speak of conceptual understanding of mathematics nor does it include any reference to creativity in mathematics. In the end, it was decided that the term *intellectually advanced mathematicians* captured the essence of those with precocity and promise, but it did not exclude individuals who were academically advanced or creatively gifted. In fact, many intellectually advanced mathematicians are academically advanced as well, so the term intellectually advanced and academically advanced are

not necessarily dichotomous. The term intellectually advanced mathematicians does not, as the term academically advanced mathematicians does, allude to a simple understanding of procedures in arithmetic.

Unlike academically advanced mathematicians, intellectually advanced mathematicians may have valleys and peaks in performance. However, intellectually advanced mathematicians may have periodic spurts of greatness in novel situations whereas academically advanced mathematicians will often be steady with little to no significant insight(s) in mathematics. Further, intellectually advanced mathematicians may have very high levels of intrinsic motivation to learn mathematics for the sake of learning mathematics. It could be said that such students are mastery-goal oriented (Phan, 2010). Conversely, academically advanced mathematicians may be extrinsically motivated by grades and report cards or performance-goal oriented (Phan). In the end, previous terms either appear to be too broad, such as gifted or talented, or appear to describe a rather different sample of gifted students such as the term mathematically capable. Intellectually advanced mathematician(s) has thus been adopted because it truly captures the emphases of this book. To reiterate, individuals such as teachers, parents, curriculum coordinators, and administrators may find that the precision and concept of intellectually advanced mathematicians intriguing. However, the suggestions and generalizations forwarded in this book do not exclude individuals with an interest in gifted and talented programs, precocious individuals, academically advanced individuals, and the like.

- **How is the term intellectually advanced mathematician(s) operationally defined?**

This is a most important question as this book unfolds. Some of this discussion was generated in the previous section. It might be implied that the term intellectually advanced is, like some of the aforementioned terms, elitist and too narrow of a term. Others may complain that the term is simply too broad of a term though once completely understood, it is not a large population. When used in reference to mathematicians, the term *intellectually advanced* is meant to encompass individuals who think in an efficient manner, think creatively, and/or have capabilities of extremely high levels of success in mathematical problem solving.

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➢ How is this book arranged?

This book is arranged somewhat sequentially. As an example, grasping the concept of intellectual advancement is requisite to understanding cognitive characteristics of intellectually advanced students. Subsequently, understanding how students are identified is significant. Therefore, the introduction sets up the reader to comprehend Chapter 1. Information contained in Chapter 1 is necessary for readers to fully grasp Chapter 2. Chapter 2 is logically followed with Chapter 3, et cetera. The chapters are detailed below:

- Introduction: Rationale for book as well as definitions
- Chapter 1: What are cognitive characteristics of intellectually advanced mathematicians?
- Chapter 2: How are intellectually advanced students in mathematics, grades K-6, identified?
- Chapter 3: What role does affect play in academic achievement?
- Chapter 4: What role does creativity play in mathematics?
- Chapter 5: Serving intellectually advanced mathematicians inside the classroom: What teaching tactics should be used?
- Chapter 6: Serving intellectually advanced students outside the classroom: What special programs exist?
- Chapter 7: What is the value of mathematical problem solving in serving intellectually advanced mathematicians?
- Conclusion

➢ What is the (educational) philosophy behind the book?

Invariably, one of the few observations that holds true in education is that there is no one way to educate everyone. This philosophy is as applicable to gifted education as it is to special education and general education. Considering that, there are approaches that work in multiple situations and approaches that are effective somewhat infrequently with intellectually advanced mathematicians. As an example, using the rather antiquated drill and practice method exclusively is not an approach that is commonly accepted in mathematics education today (National Council of Teachers of Mathematics, 2000). Theories and studies from the field of educational psychology substantiate this claim. This is because students need to have some sense of automaticity (i.e. the ability to regurgitate responses without thought) because it enables them to access cognitive energy for more complex demands such as critical thinking that is requisite for success in mathematical problem solving (Chamberlin, 2008). However, with an exclusive focus on algorithms, students may not know how to solve actual mathematics problems that are novel.

- **Are there any final admonitions?**

It is a commonly accepted practice by some researchers and theoreticians to use only recent work, e.g. last five to seven years, in literature reviews. Though high impact research that is dated may be somewhat atypical, it is naïve to suggest that these works should be altogether neglected. Similarly, it is rather egocentric to admit that only research that has been conducted in the past five to seven years is of significance to the world of gifted education and to the understanding of intellectually advanced mathematicians.

As an example, Julian Stanley's work (1975) is cited in the book on identification of intellectually advanced students in mathematics because it had perhaps the greatest influence on how identification was initially designed in mathematics gifted education. Similarly, Krutetskii's (1976) work from the late 1950s and early to mid-1960s is timeless and provides great insight for how intellectually advanced mathematicians think even though the research was conducted nearly 50 years ago.

In addition, each chapter will have a concluding section about how theory is impacted from research presented in each book. In between the introductory literature review and the concluding chapter on theory, the chapters are filled with pragmatic discussions designed to guide in-service teachers, parents, graduate students, and aspiring gifted educators such as pre-service teachers on how to adequately meet the needs of the intellectually advanced mathematicians.

* (Refer to page 6) Federal funding for gifted education has been comprised of the Jacob Javits Fellowship Grant. In the 2011, the total available funds therefore were $9,687,000 available federally (United States Department of Education, 2011). With federal government expenditures totaling 900 billion dollars in 2011 (US Government Spending, 2011), the percentage of government spending on gifted education is just over 1 dollar for every 100,000 dollars spent or 1/1000%.

My Questions as I read this book . . .

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